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NATURAL CONVECTION HEAT TRANSFER IN AIR CONFINED BY TWO VERTICAL WALLS OF DIFFERENT CORRUGATION

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Abstract A numerical study has been conducted to investigate the effects of corrugation shapes on natural convection heat transfer in a square enclosure. Two types of corrugation (vee and sinusoidal) on vertical walls of the enclosure are considered with different corrugation frequencies. The vorticity stream function formulation with the Control Volume based Finite Element Method (CVFEM) has been used to see the effect of corrugation on heat transfer for different Grashof numbers. The local and overall heat fluxes for corrugated surfaces are compared with each other and also with that for straight walls (i.e. no corrugation). The results show that the overall heat flux increases with the increase of corrugation frequency for low Grashof number but the trend is reverse for high Grashof number. For both of Grashof numbers and corrugation frequencies the total heat flux is higher for sinusoidal corrugation than that for vee corrugation. At low Grashof number the total heat flux through corrugated walls is higher than that through straight wall for all corrugation frequencies. At high Grashof number the total heat flux of corrugated surface is higher for low corrugation frequency and lower for high corrugation frequency than that of straight walls. For all cases the sinusoidal corrugation shows higher heat flux than that of vee corrugation.

Keywords: Sinusoidal corrugation, Vee corrugation, Convection heat transfer, Corrugation frequency.

INTRODUCTION

The buoyant force causes the flow of the fluid, which is the consequence of temperature gradient. The flow of the fluid transfers internal energy stored in fluid elements and it is termed as convection heat transfer. If the convection current occurs only by thermal expansion of the fluid particle, then it is called natural convection and the corresponding heat transfer is termed as natural convection heat transfer. But the intensity of the mixing of the fluid is generally less in natural convection, and consequently the heat transfer coefficient in natural convection is lower than that in forced convection. Also the shape of the heat transfer surface influences the behavior of the flow.

Several researchers carried out studies on convective heat transfer with corrugated walls; but they have only considered convection from a horizontal lower corrugated plate to an upper cold flat plate. None of them performed an experiment on convection heat transfer with vertical hot and cold corrugated plates, and this motivates the present study. [Chinnappa, 1970] carried out an experimental investigation on natural convection heat transfer from a horizontal lower hot vee-corrugated plate to an upper cold flat plate. He took data for a range of Grashof numbers from 10^4 to 10^6. The author noticed a change in the flow pattern at Gr = 8×10^4, which he concluded was a transition point from laminar to turbulent flow. [Randall *et al.*, 1979] studied local and average heat transfer coefficients for natural convection between a vee-corrugated plate (60^o vee-angle) and a parallel flat plate to find the temperature distribution in the enclosed air space. From this temperature distribution they used the wall temperature gradient to estimate the local heat transfer coefficient. Local values of heat transfer coefficient were investigated over the entire vee-corrugated surface area. The author recommended a correlation in which the heat flux was 10% higher than that for parallel flat plates.

[Zhong *et al.*, 1985] carried out a finite difference study to determine the effects of variable properties on the temperature and velocity fields and the heat transfer rate in a differentially heated, two-dimensional square enclosure. [Nayak and Cheny, 1975] considered the problem of free and forced convection in a fully developed laminar steady flow through vertical ducts under the conditions of constant heat flux and uniform peripheral wall temperature. [Chenoweth and Paolecci, 1986] obtained steady-state, two-dimensional results from transient Navier-stokes equations given for laminar convective motion of a gas in an enclosed vertical slot with large horizontal temperature differences. In the present investigation the effects of corrugation shapes of the vertical walls, corrugation

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frequency and Grashof number on local and overall heat transfer rates, and velocity and temperature distribution have been examined both qualitatively and quantitatively.

PROBLEM DESCRIPTION

The problem schematic is shown in Fig. 1(a-b). The top and bottom walls of the enclosure are insulated and the left and right vertical walls are either vee-corrugated or sinusoidal corrugated. The left and right walls are kept at constant temperature. The temperature of the left wall is T_h and that of the right wall is T_c, where $T_h > T_c$. The characteristic length of the square enclosure is L. The origin of the X-Y co-ordinate system is located at the left bottom corner of the cavity.

MATHEMATICAL MODELLING

The Navier-Stokes equations for two-dimensional, incompressible flow with constant properties in cartesian co-ordinates can be written as follows:
Continuity equation,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

x-momentum equation,

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+S^u \qquad (2)$$

y-momentum equation,

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+S^v \qquad (3)$$

In the above equations, u and v represent the velocity components in the x and y directions respectively, and p is the pressure. The source terms S^u and S^v consider the other body and surface forces in the x and y direction, respectively and ν is the kinematic viscosity. By introducing appropriate buoyancy term in the momentum equations the natural convection heat transfer problem can be solved by these equations.

By differentiating equations (2) and (3) with respect to y and x, respectively and then subtracting the results of the former from the later, a single vorticity transport equation can be obtained:

$$u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=\nu\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)+\left(\frac{\partial s^v}{\partial x}-\frac{\partial s^u}{\partial y}\right) \qquad (4)$$

where ω is the vorticity, defined as

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \qquad (5)$$

Upon defining the stream function, ψ as

$$\frac{\partial \psi}{\partial y}=u \quad \text{and} \quad -\frac{\partial \psi}{\partial x}=v$$

and substituting into Eq (5) the Poisson equation relating ω to ψ may be obtained as

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}+\omega=0 \qquad (6)$$

The details for implementing ω and ψ conditions are available in [Husain, 1987].

Assuming the properties to be constant, other than the density variation in the buoyant forces, the Boussinesq approximation [Bejan, 1984], which consists of retaining only the variations of density in the buoyancy terms, may be used on equation (4) which results in

$$u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=\nu\left(\frac{\partial^2 \omega}{\partial x^2}+\frac{\partial^2 \omega}{\partial y^2}\right)+g\beta\frac{\partial T}{\partial x} \qquad (7)$$

The energy transport equation for two-dimensional incompressible flow with constant properties [Patankar, 1980] can be written as,

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (8)$$

where α is the thermal diffusivity of the fluid.

Equations (4) to (8) can be normalized by introducing the following non-dimensional quantities.

$$X=\frac{x}{L},\quad Y=\frac{y}{L},\quad U=\frac{uL}{\nu},\quad V=\frac{vL}{\nu}$$

$$\Omega=\frac{\omega L^2}{\nu},\quad \Psi=\frac{\psi}{\nu},\quad \theta=\frac{T-T_c}{T_h-T_c}$$

to yield

$$U\frac{\partial \Omega}{\partial X}+V\frac{\partial \Omega}{\partial Y}=\frac{\partial^2 \Omega}{\partial X^2}+\frac{\partial^2 \Omega}{\partial Y^2}+Gr\frac{\partial \theta}{\partial X} \qquad (9)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (10)$$

where Gr and Pr are the Grashof and Prandtl numbers [Ozisik, 1985] respectively and defined as:

$$Gr=g\beta(T_h-T_c)L^3/\nu^2 \qquad (11)$$

$$Pr=\nu/\alpha \qquad (12)$$

Here the parameters g, β and α represent the acceleration due to gravity, the coefficient of thermal expansion and the thermal diffusivity of the fluid, respectively.

The boundary conditions of the problem are as follows:

(i)	$\frac{\partial \psi}{\partial y}=0$	at all walls
(ii)	$\frac{\partial \psi}{\partial x}=0$	at all walls
(iii)	$\Psi=0$	at all walls
(iv)	$\theta=1$	at left wall
	$\theta=0$	at right wall

(v) $$\left(\frac{\partial \theta}{\partial Y}\right)_{Y=0} = \left(\frac{\partial \theta}{\partial Y}\right)_{Y=1} = 0$$

METHODOLOGY

The calculation domain is first discretized as in [Ali and Husain, 1992]. Following the domain discretization, the integral formulation of the relevant transport equations [Husain, 1987] is imposed on each control volume [Baliga and Patankar, 1980] of the overall region. The solution of the governing equations is obtained iteratively and once convergence of the equations has been achieved the following quantities are calculated.

The local Nusselt number along the hot wall,

$$Nu_Y = \frac{\dot{q}'' L}{\kappa (T_h - T_c)} = -\frac{\partial \theta}{\partial N} \tag{13}$$

The dimensionless total heat flux at the hot wall,

$$Q = -\int_{Y=0}^{Y=1} \frac{\partial \theta}{\partial N}\, dS(Y) \tag{14}$$

Where S is the dimensionless distance measured along the corrugation of the wall and N is the dimensionless distance measured normal to the same. In equation (13) $\dot{q}''$ is the heat flux per unit length at the hot wall and κ is the thermal conductivity.

Table 1: Summary of computational runs

Corrugation Frequency (CF)		Grashof number (Gr)
Vee cor.	Sinusoidal cor.	
1	1	10^3
		10^4
		10^5
2	2	10^3
		10^4
		10^5
3	3	10^3
		10^4
		10^5

RESULTS AND DISCUSSION

In this investigation the total heat transfer through the enclosure, vertical velocity and temperature distributions at the horizontal mid-plane, and local Nusselt number along the hot wall are examined with respect to Grashof numbers 10^3, 10^4 and 10^5 for different shapes of corrugated walls and corrugation frequencies. The total heat flux of both corrugation shapes are compared with that of straight wall. The corrugation amplitude is fixed at 5 percent of the enclosure height for all runs, where the amplitude "A" is defined as half of the horizontal distance measured from the left extremity of the left wall to its right extremity as shown in Fig.1. Henceforth, the left and right extremities of the hot wall will be referred to as the "trough" and "peak", respectively. Here grid size 31x31 was considered for all runs. The summary of computational runs has been shown in Table-1.

Effects of Corrugation Shapes on Heat Flux

Table-2 shows the effects of corrugation shapes on the total heat flux (Q) with different Grashof numbers. For easy understanding the results in Table-2 are discussed under the following titles: (i) Heat flow through corrugated and straight walls, and (ii) Heat flow through different corrugated walls.

Table 2: Total heat flux (Q) for corrugated and Straight (St.) walls with different Gr.

Gr	Q for CF = 1		Q for CF = 2		Q for CF = 3		St. wall
	Vee cor.	Sinus-oidal	Vee cor.	Sinus-oidal	Vee cor.	Sinus-oidal	
10^3	1.126	1.186	1.132	1.207	1.135	1.214	1.121
10^4	2.295	2.339	2.271	2.321	2.238	2.301	2.270
10^5	4.837	4.850	4.753	4.783	4.573	4.624	4.724

(i) Heat flow through corrugated and straight walls – Table-2 shows that for both corrugations Q increases continuously with the increase of Corrugation Frequency (CF) for Gr=10^3 whereas for Gr=10^4 and 10^5, Q decreases with the increase of CF. For Gr=10^3 the continuous increase of Q with CF may be attributed to the enhancement of surface area but the decrease of Q for higher Gr may be explained as the retardation of flow due to increased waviness of the corrugation. This behavior may be explained by asserting that at high Gr the fluid velocity increases near the peaks but drops near the trough as the boundary layer tends to separate. Thus the fluid fails to maintain close contact near the trough of the corrugation resulting in decreased convection heat transfer, where as for Gr=10^3, the low vertical velocities thus generated enable the fluid to maintain better contact with the corrugated wall. Thus with the higher CF the corresponding enhancement of heat transfer surface area leads to increased total heat flux at low Gr. For the case of high Gr, the lower velocities and consequent decrease in convective heat transfer at the troughs more than offsets the increased surface area.

(ii) Heat flow through different corrugated walls – In general it can be found from Table-2 that the overall heat flux through sinusoidal corrugated walls is higher than that through vee corrugated walls for all Grashof numbers and CFs. For Gr=10^3 the increasing rate of Q is higher in sinusoidal corrugation with CFs. This can be explained by the fact that sinusoidal corrugation increases the heat transfer area more than that of vee corrugation and the low vertical velocity due to low Gr enables the fluid to maintain better contact with larger heat transfer area resulting higher total heat flux. For Gr=10^4 and 10^5, Q decreases with the increase of CF caused by the increased waviness of the corrugation.

However the increase in Q for sinusoidal corrugation can be explained by asserting that the smoothness of the curvature at both the peaks and troughs enables the fluid to maintain better contact with the sinusoidal corrugation.

Effects of CF on Local Nusselt Number

The decreasing nature in convection heat transfer through corrugated walls is evident upon referring to Fig. 2(a-b) where it may be observed that the local Nusselt number which is identical to the dimensionless local heat flux attains minimum values at the troughs of the corrugation. It can be noted from this figure that there is a significant increase in local Nusselt number at the peaks of the corrugation and decrease of the same at the troughs. The reason is that the peaks cause the fluid to come in contact more intimately with the surface resulting in large convection heat transfer and consequently the local Nusselt number increases. Fig. 4 also shows that the peak value of Nu_Y decreases with increasing vertical distance along the corrugated wall. This may be explained by the fact that the colder fluid collects at the bottom-left corner of the enclosure creating a large temperature gradient with the hot wall and as it moves up and receives heat, the temperature gradient decreases causing the decrease in local Nusselt number.

Effects of CF on Vertical Velocity and Temperature Distribution

Figs. 3 and 4 reveal the effect of corrugation on vertical velocity distribution at the horizontal mid-plane for $Gr=10^5$ and 10^3 respectively where CF=3. Fig. 3 indicates that for $Gr=10^5$ the peak value of the vertical velocity decreases for corrugated surface and becomes lowest for vee corrugated surface. This trend can be explained by examining Fig. 5 which indicates that the temperature gradient is lower for sinusoidal corrugated surface causing a lower buoyant force and hence a lower vertical velocity. Similarly the temperature gradient is lowest for vee corrugated surface and hence causes lowest buoyant force and a lowest vertical velocity. Because of this lower velocity, the strength of convection heat transfer decreases with corrugated surfaces which has been shown in table-2. But in Fig. 4 the vertical velocity increases with corrugated surfaces, which leads to an increase in overall heat transfer.

Effects of Gr on Total Heat Flux

Figs. 6(a-b) show the variation of Q as a function of Gr for corrugated and straight surfaces for CF=1 and 3, respectively. Fig. 6a shows that for CF=1 the increment of Q increases with Gr when compared between the corrugated and straight walls which is caused by the enhancement of the heat transfer surface area. Reverse trend can be found when compared between the increment of Q through sinusoidal and vee corrugated walls. More clearly, for all Grashof numbers the sinusoidal wall has higher Q and the increment of Q for sinusoidal case decreases with the increase of Gr. This behavior can be explained by asserting that the fluid fails to maintain close contact to the troughs resulting in lower increment of Q. Fig. 6b shows that the variation of Q between vee corrugated and straight walls is greater for higher Gr than that between sinusoidal and straight walls. The curves of vee corrugated and straight walls cross at around $Gr=10^3$ indicating a trend reversal which was discussed earlier. On the other hand, the variation of Q between sinusoidal corrugated and straight walls is greater for lower Gr than that between vee corrugated and straight walls. Nearly constant variation of Q can be found between vee and sinusoidal corrugated walls for lower Gr. This behavior can be explained by asserting two competing effects: the enhancement of wall surface area and the early separation of flow for sinusoidal corrugation. Specifically the increase in wall surface area tends to enhance the overall heat transfer while the separation of flow tends to reduce the convection transport of energy.

CONCLUSION

The above analysis shows that the overall heat transfer rate for low Grashof number increases for corrugated surfaces but the trend is reversed for high Grashof number. For low Grashof number the sinusoidal corrugation shows higher increment of Q than that of vee corrugation when compared with straight wall. Moreover, for both of Grashof numbers and corrugation frequencies the sinusoidal corrugation shows higher heat flux than the vee corrugation. The local Nusselt number increases significantly at the peaks and attains minimum values at the troughs and the peak values of Nu_y decrease with the increase of vertical distance along the corrugated wall. It is also observed that for high Grashof number the temperature gradient becomes lower with corrugated surface causing the lower vertical velocity of the fluid but the trends is reversed for low Grashof number. It can be pointed out that the decreased nature of heat transfer rate for corrugated surface may be applied in practical situation where heat transfer reduction is desired across large temperature differences.

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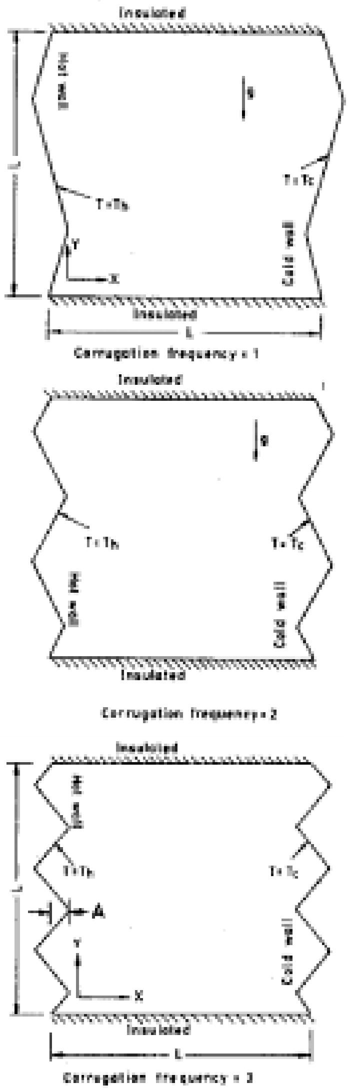

Fig.1a Schematic of the vee corrugated domain.

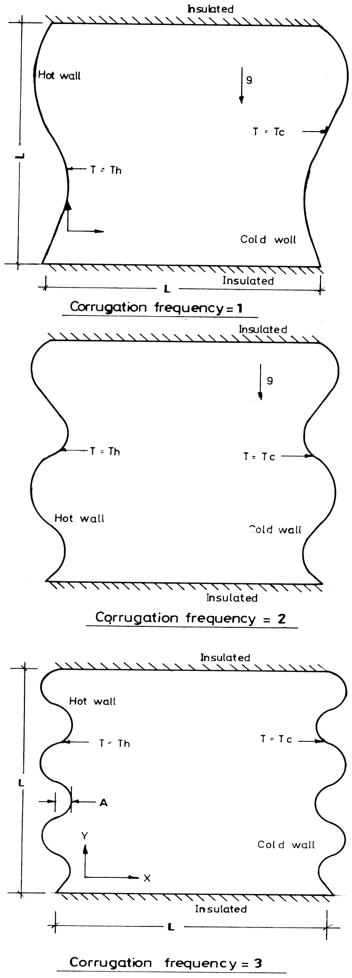

Fig.1b Schematic of the sinusoidal corrugated domain

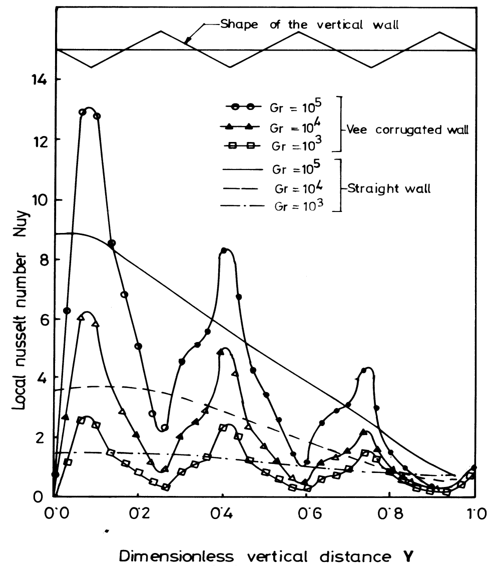

Fig.2a Local Nusselt number distribution on the hot wall (Vee corrugated).

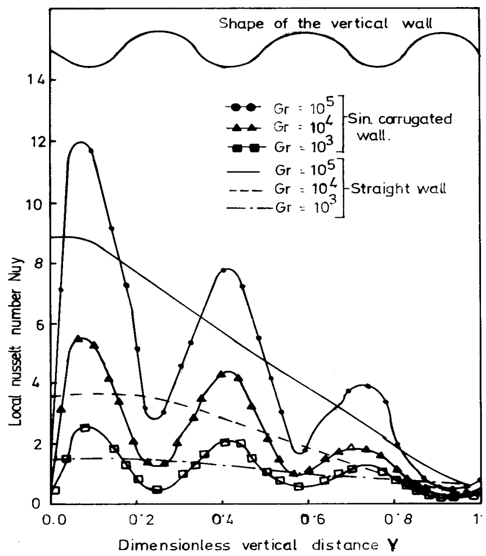

Fig.2b Local Nusselt number distribution on the hot wall (Sinusoidal corrugated).

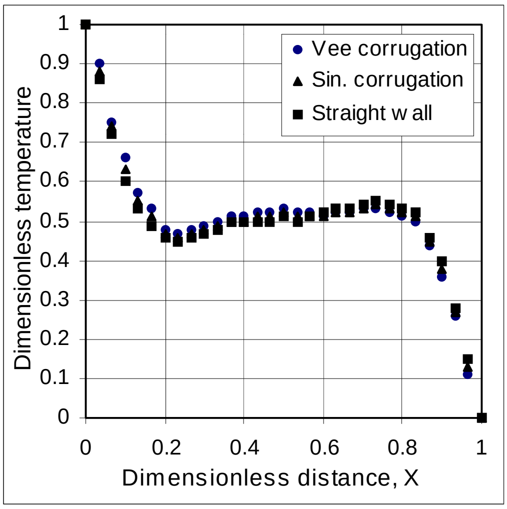

Fig.5 Temperature distribution at the horizontal midplane for Gr=10^5 and CF=3.

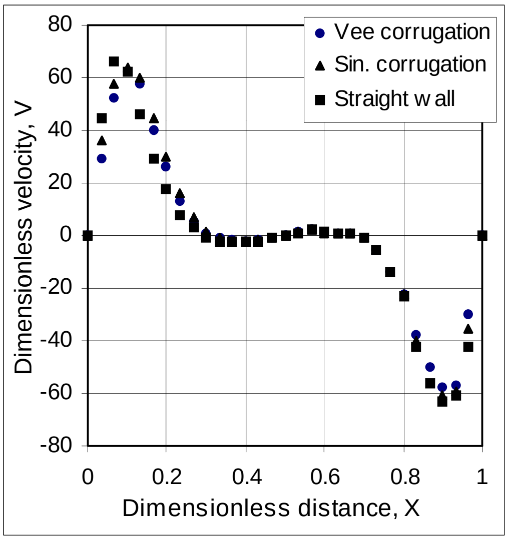

Fig.3 Vertical velocity distribution at the horizontal midplane for Gr=10^5 and CF=3.

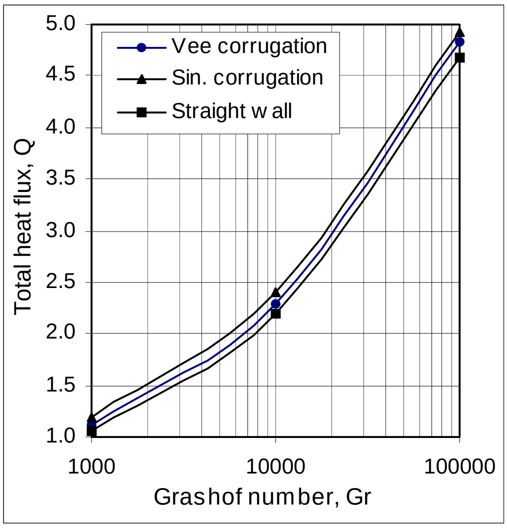

Fig.6a Variation of Q with Gr for CF=1.

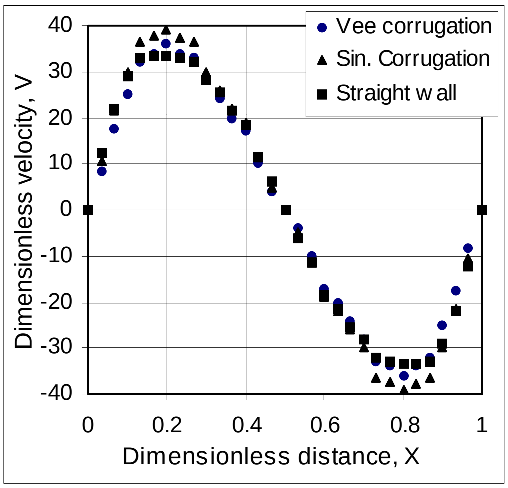

Fig.4 Vertical velocity distribution at the horizontal midplane for Gr=10^3 and CF=3.

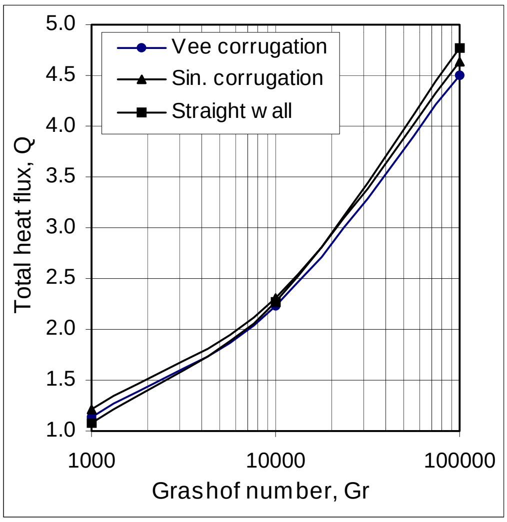

Fig.6b Variation of Q with Gr for CF=3.